

LAB III. SIMPLE PENDULUM

1. Objectives

This experiment focuses on studying the properties of a simple pendulum as a function of the object's mass, the length of the string and the initial angle of the pendulum. For all trials, assume the mass of the string to be negligible and make sure that the initial angles are less than 20 degrees. By recording videos and analyzing the motion using the Tracker app, determine the period and amplitude of the oscillation, as well as the kinetic and potential energy at specific times. Calculate the gravitational constant.

2. Material (see **Fig. 1**)

- 2 Ping-pong ball/ Golf ball/ Tennis ball/Playdough etc.
- String
- A Meter stick/ruler
- A Scale (Kitchen scale)
- A Glue gun
- 2 suction cups with hooks

Fig. 1: An example list of materials used in this lab.

3. Theory

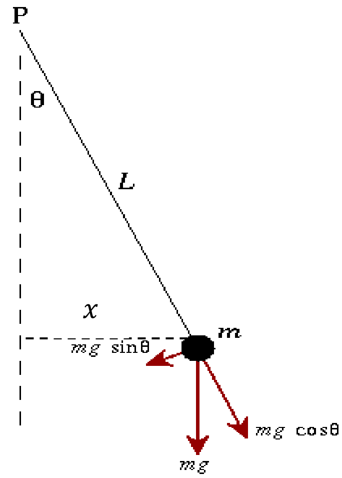

Fig. 2: A schematic example of simple pendulum

A simple pendulum consists of a mass *m* hanging from a fixed pivotal
point *P* with a massless string of length *L*. When the pendulum is released
from a position with an initial angle θo, it will swing back and forth with a certain periodicity.
Writing down the equation of motions:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin(\theta) = 0. \quad (1)$$

If the angluar displacements are small, one can make the approxmation $\sin(\theta) \approx \theta$ and get the equation for simple harmonic motion,

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0. \quad (2)$$

Considering $\theta \approx \frac{x}{L}$, **Eq. 2** can be simplified as,

$$\frac{d^2x}{dt^2} + \frac{g}{L}x = 0. \quad (3)$$

The solution can be written as,

$$x = x_0 \sin(\omega t + \phi), \quad (4)$$

where x_0 is the amplitude of the motion, ω is the angular velocity which is $\sqrt{\frac{g}{L}}$ and ϕ is the phase of motion. The period of the motion can be found from $T = 2\pi/\omega$ as,

$$T = 2\pi\sqrt{\frac{L}{g}}. \quad (5)$$

4. Lab Setup preparation

There are several instructional videos on the preparation of the lab. You need to watch

(1) **Preparing the Meterstick** video (https://www.youtube.com/watch?v=67j4kJOiAQI)

(2) **Phone Setup** video (https://www.youtube.com/watch?v=nHfeejFBe28)

(3) **Tracker Tutorial** video (https://www.youtube.com/watch?v=BxplFubEVzQ)

Please make sure that you follow the detailed instructions to avoid potential errors in data analysis before attempting to perform the experiment. Here, important notes on the setup preparation are described in detail.

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A detailed video instruction on how to prepare and perform simple pendulum motion can be found at,

https://www.youtube.com/watch?v=elFAgPDr-LI&t=11s

and follow the below steps to prepare the set-up:

4.1 Prepare the hanging object. We recommend using playdough which allows you to vary the mass continuously. One example on how to hang a playdough object is provided in the instruction video, where you first tie the string to a washer, then cover it with playdough until you get to the desired mass.
4.2 Alternatively, you can hang a ball (ping-pong, tennis, golf ball, etc.). Use a glue to attach the string to the object. **Please refer to the video instruction on how to use the glue gun before attempting to work with it. Serious injuries could occur if you do not follow the exact procedure.**
4.3 Wind the string over the suction hook

4.4 Measure the length of the string, from the suction hook to the ball.

4.5 Measure the total mass of the hanging object.

4.6 Firmly press the suction hook onto a surface (table, wall, etc.)

A sample setup is shown in Fig. 3. Note that we have a meterstick attached to the table using double sided tape. The meterstick or ruler is necessary for calibration in video analysis.

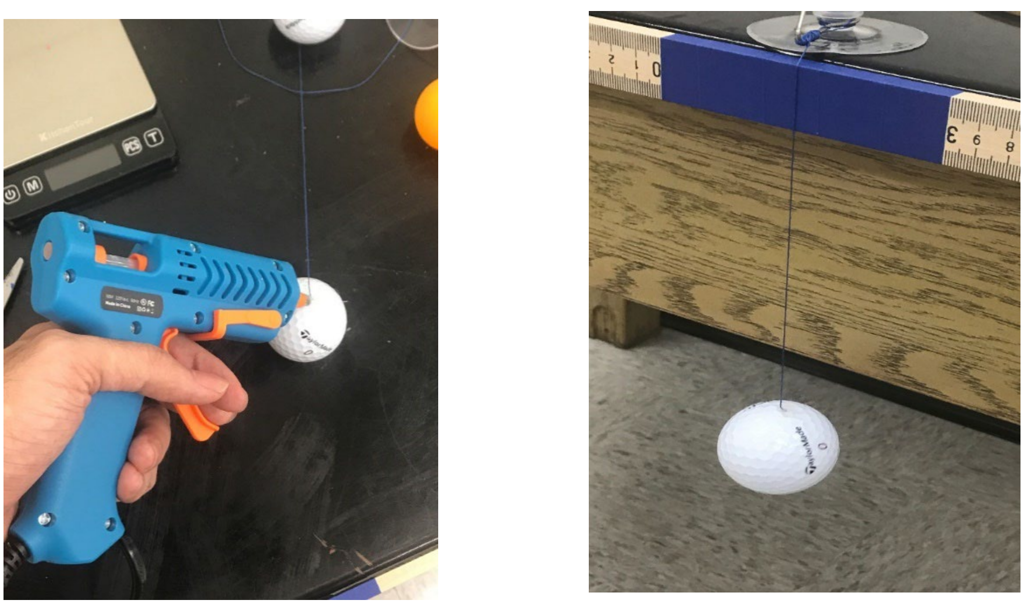

Fig. 3: Using a glue gun to attach a string to a golf ball (left) and hanging it from a suction hook (right)

5. Experiment & Data Analysis Procedure

In the first part, the focus will be on the effects of initial angle and mass of the hanging object. You will find the period of the motion and then see whether these two factors affect it. In the second part, you need to fit a sinusoidal function to the displacement of the object and then find the period. In the end, you can estimate the earth gravitational acceleration and compare it with the exact value.

5.1 Dependence on initial height

5.1.1 *Perform the experiment*: Record five separate experiments with different initial heights.

5.1.2 *Initial angle*: Use "Tracker" to estimate the initial angle the motion. Please watch the video instruction for the current lab to learn how to measure angles.

5.1.3 *Average Period*: When recording the experiment, make sure to record long enough to capture 5 full periods of the motion. Measure the time when the ball finishes each period and find the average of all periods.

5.1.4 *Comparison with theory*: Compare the average periods at different initial heights and find the percent error with respect to the theoretical value in **Table. 1**.

5.2 Dependence on mass

5.2.1 *Different mass*: Vary the mass of the object and fill in the **Table. 2**. This can be done by either using playdough as the hanging object or using balls with different masses.

5.2.2 *Initial angle*: When performing the experiment, make sure the initial angle is reasonably small so that $\sin\theta \approx \theta$.

5.3 Dependence on Length

5.3.1 *Different length*: Keep the mass of the object constant and vary the length of string by winding the string over the suction hook. Make sure to measure the length before performing the experiment.

5.3.2 *Perform the experiment*: Record five separate experiments with different lengths of string. It is recommended that the starting length is at least 30 cm.

5.3.3 *Length in Tracker app*: Estimate the length of the string from the Tracker app and compare it with the value you measured before performing the experiment.

5.3.4 *Period vs length*: Find the average period the same way as in Part A and fill in the second column in **Table. 3**. Then, fit a sinusoidal function $A\sin(\omega t + \phi)$ according to **Eq. 4** to the recorded angles, find the period and fill in the next column.

5.3.5 *Gravitational acceleration*: The squared of Eq. 5 would be $T^2 = \frac{2\pi L}{g}$. Plot the length versus the average measured period and fit a quadratic function. Estimate g from the fitted parameters.

5.3.6 *Initial angle*: Perform the experiment again but with noticeably higher initial angle. Record five separate experiments and estimate g as in step 5.3.5.

Lab III Worksheet

Name of the Student: ______________________ **Date: _____________________**

Q.1 Dependence on initial height

Length of the string: ----------------

Theoretical value for period according to **Eq. 5**: ---------------

Table. 1

Trial	θ_0	Average Measured Period	Percent error
1			
2			
3			
4			
5			

Q.1.1 Do you expect the period to be independent of θ_0? Explain.

Q.1.2 Can you notice a systematic change in the percent error? Explain.

Q.2 Dependence on mass

Perform the experiment with three different masses of playdough.

Table. 2

Mass (kg)	Average Measured Period	Percent error

Q.2.1 Does the period depend on the mass? Explain

Q.3 Dependence on Length

Adjust the length of the string by winding the string around the rod. Fill in the table in the increasing order in the length. Perform the experiment at least 5 times for each length, and record the average period (T) in the second column.

Table. 3

L	Average Measured T	Fitted T	Percent error

Q.3.1 Estimate the length of the string from the Tracker and compare with the measured length from the ruler. Does the Tracker underestimate or overestimate the length? Explain.

Q.3.2 Fit a sinusoidal function to the θ and find the parameters. What are the physical meaning of parameters according to the **Eq. 4**.

Q.3.3 What is the unit of θ in **Eq. 2**?

Q.3.4 Plot the length values (L) versus the average measured period. Fit a quadratic function to your data points.

Q.3.5 What are the parameters of your fitted function? Comparing the fitted function with **Eq. 1**, what are the physical meanings of the parameters?

Q.3.6 Do you overestimate or underestimate g? What are the sources of error?

Perform the experiment 5 times again but at a noticeably higher initial angle.

Q.3.7 Estimate *g* according to *Q.3.4* to *Q.3.6*. Does your estimation of *g* improve or worse compared to previous questions? Explain

Q.3.8 Explain at least one way to modify the experiment and achieve better agreement with the correct *g* value.

Troubleshooting

- Given that the velocity of the object is relatively small, the slow-motion video recording is not necessary. However, we still recommend using at least 60 frames per second.
- For each trial, record the motion for up to 10 seconds. The video should be long enough that the object finishes a full period at least 5 times.
- Try to use spherically shaped objects. The center of mass would then be at the center for a uniform object.