

LAB VII. RULER ROTATION

1. Objectives

The purpose of this lab is to understand rotational motion and analyze angular velocity and acceleration. You will study the rotation of a ruler along an axis and track the angular changes at different radius values from the axis of rotation. Then, based on torque equations, you would be able to estimate torque and eventually moment of inertia.

2. Materials (see **Fig. 1**)

- A meterstick/ruler for calibration
- Wood Ruler with binder holes
- Poster strip tape
- A smartphone
- A smartphone tripod
- Playdough

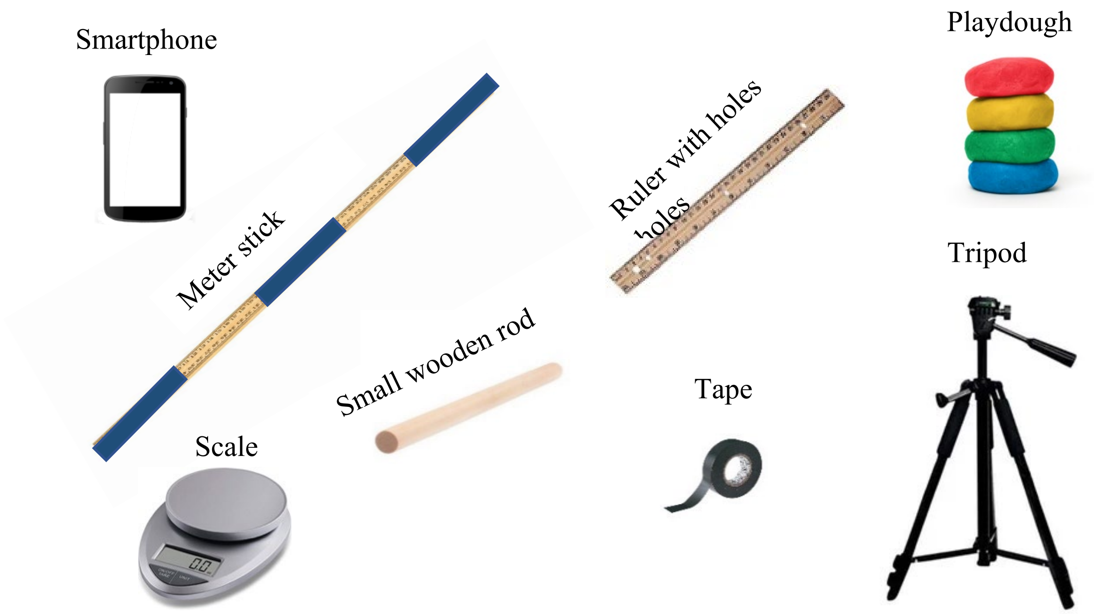

Fig. 1: An example list of materials used in this lab.

3. Theory

3.1 Rotation

In rotational motion can be analyzed in a same fashion as the translational motion. Velocity and acceleration become angular velocity and angular acceleration and are defined by,

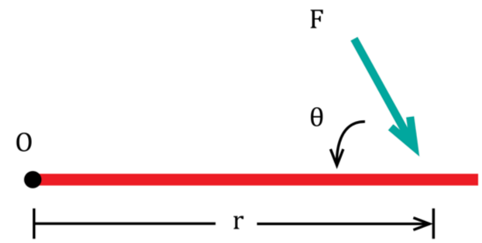

Fig. 2 Force applied at angle θ, at a radius r from the axis of rotation.

$$\omega = \frac{d\theta}{dt}, \tag{1}$$

$$\alpha = \frac{d\omega}{dt}, \tag{2}$$

where ω and α are angular velocity and acceleration, respectively. For a system that rotates about the axis O with a radius R, the translational velocity and acceleration, also called linear velocity and acceleration can be calculated as,

$$v = R\omega, \tag{3}$$

$$a = R\alpha. \tag{4}$$

3.2 Torque

Torque is a measure of how an acting force on an object causes it to rotate. It is defined with regard to the axis of rotation O, and is dependent on the magnitude of the force of F and the angle θ,

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = rF\sin(\theta). \tag{5}$$

There would be no torque if force is parallel to r ($\theta = 0$) and torque would be maximum if the applied force is perpendicular to r ($\theta = 90°$)

In rotational motion, angular acceleration (α), the rate that angular velocity changes, can be related to τ according to:

$$\boldsymbol{\tau} = I\boldsymbol{\alpha}. \tag{6}$$

This is equivalent to Newton's 2^{nd} law for linear motion $\boldsymbol{F} = m\boldsymbol{a}$. Note that I is called **moment of inertia** which depends on mass (m), shape of the object and the axis of rotation. For a general shape object rotating along the z axis, where masses of point particles (m_i) are at the distance r_i with respect to the axis of rotation, the moment of inertia can be calculated from,

$$I = \sum_i^n m_i r_i^2. \tag{8}$$

In this experiment, a ruler is hanged from one end vertically. This means, torque due to the gravitational force will cause the ruler to rotate once the other end of the ruler is lifted. Writing down equations of motion using **Eq. 5** and **6** results in,

$$I\ddot{\theta} = -\frac{mgL}{2}\sin\theta\,, \tag{9}$$

where L is the length of the ruler (assuming the axis of rotation is at one end of the ruler) and $\frac{L}{2}$ is where the center of mass of the ruler is located (assuming the ruler is uniform).

4. Lab Setup Preparation

There are several instructional videos on the preparation of the lab. You need to watch

(1) **Preparing the Meterstick** video

https://www.youtube.com/watch?v=tPTcO0XJC48&t=25s

(2) **Phone Setup** video

https://www.youtube.com/watch?v=SGu5YLBJyb8

(3) **Tracker Tutorial** video

https://www.youtube.com/watch?v=OFL3oqqk8Lk

Please make sure that you follow the detailed instructions to avoid potential errors in data analysis before attempting to perform the experiments.

A detailed video instruction on how to prepare and perform Friction experiment can be found at :

https://www.youtube.com/watch?v=Zv3e8piYbP8

4.2 Axis of rotation

4.2.1 Use a circular/square shape rod as an axis of rotation and fix it to a table using tapes.

4.2.2 Make sure there is at least 3 cm distance between one end of the rod and the edge of the table.

4.2.3 Then hang the binder hole ruler from a hole at either ends of the ruler from the rod.

4.2.4 The radius of rod should be small enough compared to the ruler hole.

4.2.5 Lift one end of the end of the hanged ruler and release it. If you notice significant friction (rotation stops after one cycle), use a smaller radius size of the rod.

4.3 Alignment and tracking

4.3.1 Rotation of the ruler should occur in a plane parallel to the camera of the smartphone.

4.3.2 If ruler moves toward or away from the smartphone, prevent it by taping the two sides of the ruler on the rod (included in the video instruction)

4.3.3 Add a circular shape playdough on the ruler to assist the tracking analysis.

4.3.4 Make sure to measure the mass of any added playdough to the ruler

5. Experiment & Data Analysis Procedure

5.1 *Experiments*: Release the ruler at an arbitrary angle, higher than 30 degrees. Record at least 5 videos of the ruler rotation.

5.2 *Coordinate system:* Move thee coordinate reference to the axis of rotation.

5.3 *Tracking:* Track the attached playdough on the ruler, at three different radius values from the axis of rotation.

5.4 *Velocity:* Plot the angular velocity (ω) and linear velocity (v) as a function of time. Then you need to investigate the relationship between them and see if they follow the **Eq. 3**.

5.5 *Acceleration:* Perform the same analysis now for angular acceleration (α) and linear acceleration (a). Does it follow the **Eq. 4**?

5.6 *Averaging:* Perform 5-point and 7-point averaging for the α. This means, similar to free fall experiment, the 5-poitn average of α at a time i can be estimated as

$$\alpha_i^{(5)} = \frac{1}{5}(\alpha_{i-2} + \alpha_{i-1} + \alpha_i + \alpha_{i+1} + \alpha_{i+2}) \tag{10}$$

5.7 *θ dependence:* Plot $\alpha^{(5)}$ and $\alpha^{(7)}$ as a function of θ. Explain how different they are compared to direct estimation of α dependence on θ.

5.8 *Fitting and Parameters:* Fit a sinusoidal function and explain the physical meaning of the parameters in the fitting.

5.9 *Moment of Inertia:* Do steps 5.7 and 5.8 for at least 5 different measurements. What is your estimation of the moment of inertia?

5.10 *Theoretical estimation:* What is the theoretical estimation of moment of inertia for uniform rectangular ruler? Does that estimation agree with your experimental analysis? Explain

5.11 *Extra mass:* Perform steps 5.7 to 5.10 now with an added mass to the ruler. In the first part of the worksheet, you are asked to add the mass to the center of the ruler and in second part, to the bottom end of the ruler. Fill in both tables and explain how moment of inertia changes in these two scenarios.

Lab VII Worksheet

Name of the Student: ______________________ **Date:** ______________________

Q.1 Angular and linear velocity

Q.1.1 According to **4.3**, you need to track the motion of 3 different radiuses from the axis of rotation. Plot ω for each radius in one graph. Explain the any differences or similarities among them. **.**

Q.1.2 Plot linear velocity (v) for all three radiuses and put them in one graph. Explain any differences or similarities. How different they are compared to ω?

Q.1.3 Estimate and plot the linear velocity from the measured ω using Eq. (3). How different they are compared to tracker estimation of linear velocity?

Q.2 Angular and linear acceleration

Q.2.1 Same as in *Q.1.1*, plot α for 3 different radius values. Explain the any differences or similarities among them.

Q.2.2 At what angles α reach the maximum value? Explain it using the definition of torque (**Eq. 5**)

Q.2.3 According to Eq. 2, explain how peak and dip in ω and v are related in your figures.

Q.2.4 Perform 5-point and 7-point averaging, similar to the analysis of free fall experiment. Explain how α behaves compared to your earlier graphs.

Q.3 Direct estimation of Moment of Inertia

Ruler mass: ---------

Q.3.1 Plot a direct estimation, 5-point average and 7-point average of α as a function of θ. Explain your observations.

Q.3.2 Fit a sinusoidal function to 5-point average and 7-point average figures.

Q.3.3 What are the physical meanings of the parameters? Explain according to Eq. 7.

Q.3.4 Perform the experiment at least 5 times. Find the moment of inertia.

Trial	**I (5-point average)**	**I (7-point average)**
1		
2		
3		
4		
5		
Average		

Q.3.5 What is the theoretical estimation of the moment of inertia, considering the ruler is uniform and is rectangular shape.

Q.3.6 How does your estimation in the table above agree with the theoretical estimation.

Q.4 Moment of Inertia with extra mass

Q.4.1 Add at least 50 g of playdough to the center of the ruler. Estimate the moment of inertia as in *Q.3.4* .

Trial	**I (5-point average)**	**I (7-point average)**
1		
2		
3		
4		
5		
Average		

Q.4.2 How does the moment of inertia changes compared to previous section?

Q.4.3 Now instead of center, add the playdough at the end of the ruler. Estimate the moment of inertia again.

Trial	**I (5-point average)**	**I (7-point average)**
1		

2		
3		
4		
5		
Average		

Q.4.4 How do you explain the differences between the *Q.4.3* and *Q.4.1*?

Q.4.5 Can you explain why using 5-point and 7-point averaging is beneficial compared to the direct approach.