

LAB VI. HOOKE'S LAW AND RUBBER BAND

1. Objectives

In this experiment, you will learn how a spring behaves when an applied force causes it to extend or compress. You will use a rubber band as an elastic object and attach different objects with varying mass to measure the stretching of the band. In the first section, you will hang a single rubber band connected to a mass and test whether it follows Hooke's law. In the second part, you will work on having multiple rubber bands and try to estimate how a combined system behaves.

2. Materials (see **Fig. 1**)

- 3 or more rubber bands
- 3 or more Suction cup with hooks
- A ruler
- A Water bottle, container, etc.
- Materials to change the weight

Fig. 1: An example list of materials used in this lab.

3. Theory

3.1 Single spring

For an object connected to a spring, there is a force proportional to the extension or compression of the spring. The force is always in the opposite direction of the length change and can be written as,

$$\boldsymbol{F} = -k(\boldsymbol{x_1} - \boldsymbol{x_0}), \tag{1}$$

where k is known as the *force constant* which depends on the properties of the spring including number of turns, thickness and the material of the spring. The linear dependence of force on the displacement is a simple approximation and it is also known as *Hooke's law*. It is important to note that **Eq. 1** fails if spring undergoes significant extension/contraction.

An example of Hooke's law is shown in **Fig. 2** where an object is initially at x_0. After extending to the new position x_1, there is a (restoring) force F in the opposite direction to which the extension occurred. Exact reverse situation happens when the object is contracted.

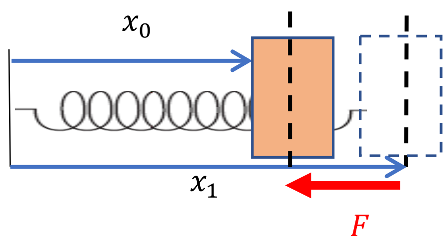

Fig. 2 Single spring attached to a mass at one end. The dotted object is after the extension occurs.

3.2 Multiple springs

A more interesting scenario is when two or more springs are connected. We will consider two different scenarios; springs are either connected in parallel or in series.

If the springs are parallel (see **Fig. 3a**), there are two forces applied on the object in the same direction. However, both springs are extended or contracted the same amount,

$$\begin{cases} F_1 = -k_1 \Delta x \\ F_2 = -k_2 \Delta x \end{cases}. \tag{2}$$

The total force F_t would just be the summation of these two forces,

$$F_t = F_1 + F_2 = -(k_1 + k_2)\Delta x. \tag{3}$$

We can now replace the two springs with an equivalent spring k_t:

$$F_t = -k_t \Delta x. \tag{4}$$

Using eq. (3) and (4), one can now find the equivalent force constant in terms of individual force constants,

$$k_t = k_1 + k_2. \tag{5}$$

This means that for any number of springs that are in parallel, the total force constant would be just the summation of the individual force constants.
For two springs in series (see **Fig. 3b**), the length change of each oscillator is different, but the total length change is a constant equal to the summation of length change in each oscillator,

$$\Delta x_t = \Delta x_1 + \Delta x_2. \quad (6)$$

Now you can use **Eq. 1** and rewrite the equation in terms of force and force constants,

$$-\frac{F_t}{k_t} = -\frac{F_1}{k_1} - \frac{F_2}{k_2}. \quad (7)$$

The mass is only connected to second spring, therefore the only force acting on the mass directly is F_2. On the other hand, if we replace the two springs with an equivalent spring, the applied force would be F_t. Therefore, these two forces are the same,

$$F_t = F_2. \quad (8)$$

Next, considering the midpoint between the spring A, when the total system is extended or contracted, there is no force acting on that point (Think about it. Does A have a mass?!),

$$F_A = F_2 - F_1 = 0. \quad (9)$$

This means all forces are equal and they can be canceled in **Eq. 7**. The final expression for two springs in series would then be,

$$\frac{1}{k_t} = \frac{1}{k_1} + \frac{1}{k_2}. \quad (10)$$

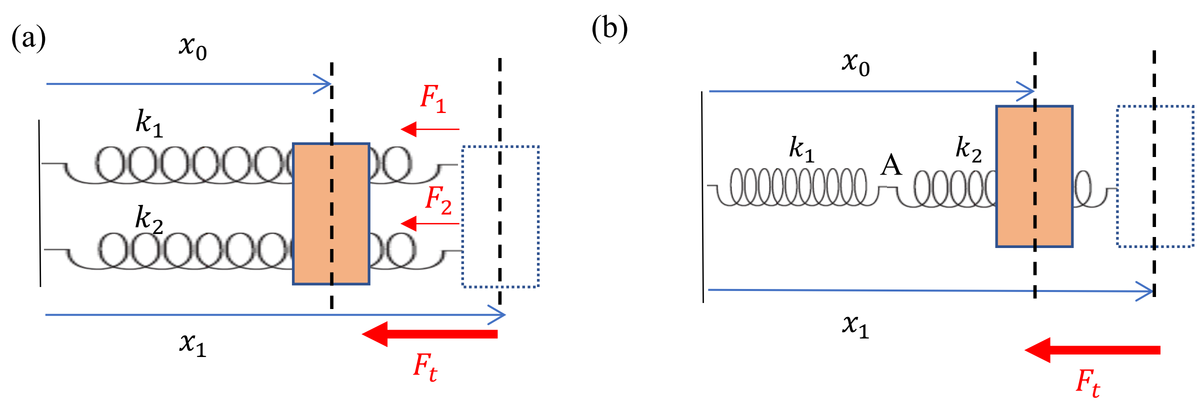

Fig. 3: Two spring systems that are in (a) parallel and (b) series. Only the extension of the system is shown where the dotted region is the object after extension.

4. Lab Setup preparation

There are several instructional videos on the preparation of the lab. You need to watch

(1) **Preparing the Meterstick** video (https://www.youtube.com/watch?v=67j4kJOiAQI)

(2) **Phone Setup** video

(https://www.youtube.com/watch?v=nHfeejFBe28)

(3) **Tracker Tutorial** video

(https://www.youtube.com/watch?v=BxplFubEVzQ)

Please make sure that you follow the detailed instructions to avoid potential errors in data analysis before attempting to perform the experiment.

A detailed video instruction on how to prepare and perform Hooke's law experiment can be found at
https://www.youtube.com/watch?v=ANfTVOZRR00

4.1 Single-cut rubber band

4.1.1 Cut a rubber band with a scissor, then tie one end to the suction cup's hook.

4.1.2 Fix the suction cup on a table or wall or any surface where it can firmly stick.

4.1.3 Tie the other end of the rubber band to the mass container. The container can be anything from a water bottle, a plastic cup, or any objects with empty volumes that you can add mass. Examples of containers are provided in the video instruction of this lab.

4.1.4 The container should be light enough to be at least 30 cm away from the ground. By adding more mass to the container, it moves toward the ground, but the container should not touch the ground.

4.2 Parallel rubber bands

4.2.1 Use another rubber band but do not cut it. Hang it from the suction cup hook at one end and at the other, use another hook connected to the container. Please watch the video instruction to see a typical example. This scenario is identical to having two cut rubber bands in parallel.

4.2.2 Now the goal is to measure the hook-to-hook distance. Similar to part A, change the mass in the container, measure the total mass, hang the container and measure the final length.

4.2.3 You can also try setting up two full rubber bands that are in series. In this case, you need three hooks, one for suction cup, one for connecting two rubber bands and the last one to hang the container.

4.2.4 Follow the same steps as in previous part.

5. Experiment & Data Analysis Procedure

5.1 Single-cut rubber band

5.1.1 *Initial length*: Record the tie-to-tie distance of the rubber band. Consider this as the initial length when no mass is attached to the rubber band.

5.1.2 *Perform the experiment*: Hang the container vertically. Then, add more mass to the container. Make sure to measure the mass before adding it to the container. Then, measure the tie-to-tie distance. Perform these steps at least 8 times and fill in the **Table. 1**.

5.1.3 *Force vs length*: Plot the force as a function of the length. Fit a linear function. It should be noted that at very short or long extensions the behavior is non-linear.

5.1.4 *Reduce the length*: Redo the experiment but start with the tie-to-tie distance half of the previous part. Repeat steps 2 to 4 and fill in the same table (columns 3 and 4)

5.1 Parallel rubber bands

5.2.1 *Perform the experiment*: Record the hook-to-hook distance between the two ends of the rubber band. Follow the steps 5.1.2 and 5.1.3.

5.2.2 *Force constant*: Find the force constant value and try to see if you can relate it to the results in the previous section. Find the force constant based on assumption that system is made of two parallel cut rubber bands and compare them with the direct measurement of the slope found in step 5.1.3.

5.2.3 *Series and parallel*: Follow the same analysis for a system made of two uncut rubber bands in series. Find the force constant directly and compare it with a similar prediction from a single rubber band.

Lab VI Worksheet

Name of the Student: ______________________ **Date: _____________________**

Q.1 Single-cut rubber band

Initial Length: ------------

Q.1.1 What is the force that extends the rubber band? Explain.

Fill in the table by measuring the applied force and length changes.

Table. 1

Initial length =		Initial length =	
Length	Force	Length	Force

Q.1.2 Plot the force as a function of length. How does the graph look like? Is it linear or quadratic? Explain the difference between the two different initial lengths.

Q.1.3 If the behavior is linear, fit a liner function and find the force constant value.

Q.1.4 How does the force constant depend on the initial length of the rubber band? Can you estimate the force constant of a rubber band with any other initial length? Explain.

Q.2 Un-cut rubber band

Initial length: ---------

Fill in the table using the same procedure as in Part A.

Table. 2

Length	Force

Q.2.1 Find the force constant. Explain how the behavior of the system differs from the single cut rubber band.

Q.2.2 A full rubber band can be assumed to act as two parallel cut rubber bands. Use **Eq. 5** to estimate the force constant of the cut rubber band from the results of *Q.1.3*.

Q.2.3 Hang another rubber band using an extra hook from the previous rubber band. According to **Eq. 10**, what is the expected force constant? Use the value you found from *Q.2.1*.

Fill in the table for the two rubber band system:

Table. 3

Length	Force

Q.2.4 Fit a linear function and find the force constant. How does it compare to value estimated from *Q.2.3*? Explain.

Troubleshooting

- Make sure the fixed point of the rubber band is not moving. Using a hook is preferred for every section of the lab
- Make sure all length measurements are done in Tracker. Measuring length with ruler has more errors.
- If you observe non-linear behavior, that is normal! Only in a specific range of length the behavior is linear.