# LAB II. PROJECTILE MOTION

## 1. Objectives

In this lab, you will study the motion of an object in two-dimensions (2D) and analyze its acceleration, velocity, and position as a function of time. The main purpose of this lab is to learn how to solve 2D motion by decomposing the motion into vertical and horizontal components. Throughout this lab, you will learn the difference between the constant velocity motion and the constant acceleration motion.

## 2. Materials (see **Fig. 1**)

- A ping-pong ball/ Golf ball/ Tennis ball/Playdough *etc*.
- Two meter-sticks/rulers
- A smartphone
- A tripod
- A surface to release a ball (can be wood, plastic, book, *etc*.)

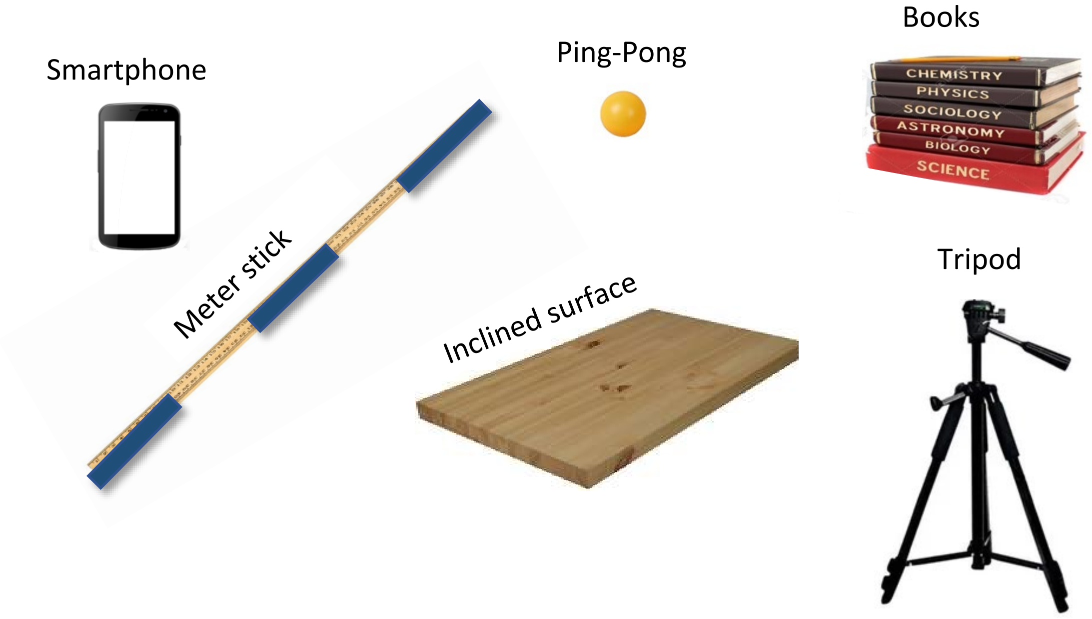

**Fig. 1**: An example list of materials used in this lab.

## 3. Theory

Projectile motion is the motion of an object thrown or projected into the air and the object is only subject to gravity in the vertical direction, assuming no air resistance. It is a 2D motion, which all motion parameters in both the horizontal (*x*-axis) and vertical (*y*-axis) directions have to be considered. In projectile motion, the motions in the x- and y-axes can be treated separately with their own accelerations, $a_x$ and $a_y$, where $a_x = 0$ and $a_y = -g$, *i.e.*, the motion in *x*-axis is a constant velocity motion while the motion in *y*-axis is a constant acceleration motion with $g$. If the initial speed of the object is $v_0$ and the initial projection angle with respect to the *x*-axis is $\theta_0$, the corresponding velocities are given by

$$\begin{cases} v_x = v_0 \cos\theta_0 \\ v_y(t) = v_0 \sin\theta_0 - gt \end{cases}, \tag{1}$$

The displacements of the object can be found as,

$$\begin{cases} x(t) = x_0 + v_0 t \cos\theta_0 \\ y(t) = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2 \end{cases}. \tag{2}$$

Note that the negative sign in the *y*-displacement expression results from the fact that the gravity acceleration pointing downward, shown in **Fig. 2**.

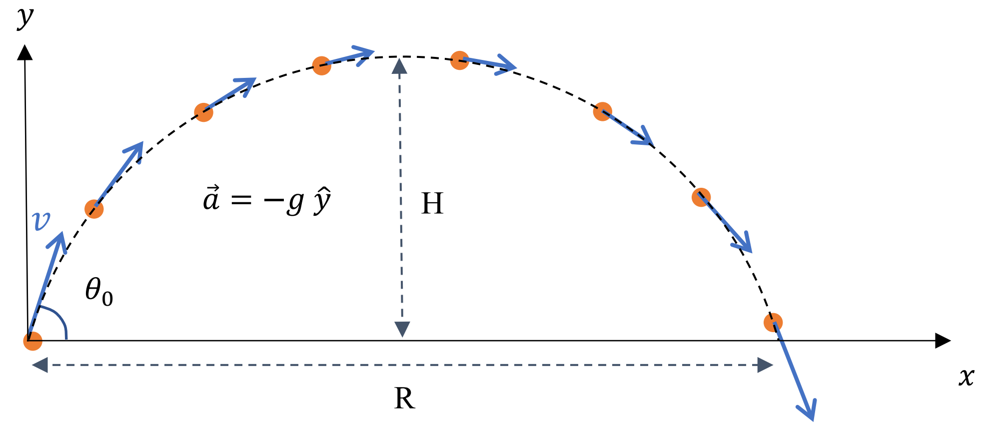

**Fig. 2** Projectile motion in 2D. The velocity vector (blue) and the maximum height ($H$) and initial angle ($\theta_0$) are shown.

## 4. Lab Setup Preparation

There are several instructional videos on the preparation of the lab. You need to watch

(1) **Preparing the Meterstick** video: https://www.youtube.com/watch?v=67j4kJOiAQI

(2) **Phone Setup** video: https://www.youtube.com/watch?v=nHfeejFBe28

(3) **Tracker Tutorial** video: https://www.youtube.com/watch?v=BxplFubEVzQ

Please make sure that you follow the detailed instructions to avoid potential errors in data analysis before attempting to perform the experiment.

A detailed video instruction on how to perform the projectile motion can be found at https://www.youtube.com/watch?v=NTlIuju_bOg.

## 4.1 Notes on preparing the 2D motion

4.1.1 You need to create a inclined surface to release a ball in a controlled manner. The inclined surface can be formed by any solid surface, such as a hardcovered book, a notebook, a laptop, a wooden board, *etc*. with a fixed angle as shown in **Fig. 3**.

4.1.1 The pivotal point of the inclined surface should be at the edge of table so that the ball will not bounce off from the table.

4.1.2 The ball will be released on the top of the inclined surface, and it moves along the inclined surface. At the end of the surface, it will be released into the air. The slope helps to guide the motion and to determine the initial projectile angle of the ball.

4.1.3 By changing the angle of the inclined surface, you can perform the 2D motion with different initial angles and initial velocities.

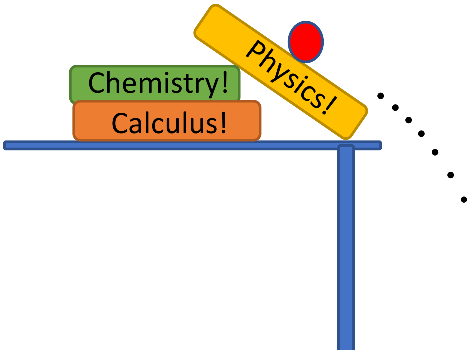

**Fig. 3** An example of setup design.

4.1.4 In the second section of the lab, you need to move the inclined surface away from the edge of the table. This will result in the bouncing of the ball from the table surface.

## 4.2 Notes on the Tracker analysis

4.2.1 Only the motion of the ball after releasing to the air or bouncing from the table's surface is what you need to track and analyze. This means that the motion of the ball on the inclined surface can be neglected.

4.2.1 The rest of the tracking steps are similar to these in free fall experiment. Make sure to select only the ball to track and use slow-mode of your smartphone.

4.2.2 Make sure to export the data for both vertical and horizontal motions. This means for every time step, you should have two positions and velocities corresponding to the vertical and horizontal components of the motion, respectively.

# 5 Experiment & Data Analysis Procedure

In the first part, you will focus on releasing the ball at the edge of a table. You are asked to change the initial velocity of the motion (*i.e.*, changing the angle of the inclined surface) and to study both horizontal and vertical components of the motion. In the second part, you will study the bouncing ball motion. You are asked to find the maximum height of travel, the total time of the travel, and the range.

## 5.1 Downward projectile ($\theta_0 < 0$)

5.1.1 *Angle measurement:* Measure the angle of the inclined surface with respect to the table. You can use a protractor to measure the angle directly or use basic trigonometry (measuring length and height of the plane) to figure out the angle, or take a photo of the inclined surface with the camera mounted perpendicular to setup and find the angle use "Tracker". Make sure the angle is constant throughout this part of the lab.

5.1.2 *Initial position*: Use pen/pencil/sharpie to mark an arbitrary location on the inclined surface where you want to release the ball.

5.1.3 *Perform the experiment*: Having the smartphone and rulers ready, hold the ball on the mark, remotely turn on smartphone recording, then release the ball. The ball should move straight downward and does not bounce from the table. Repeat these steps at least 5 times but with the same initial heights on the surface. Here, initial height is with respect to the table and not the ground.

5.1.4 *Track the ball*: Track the object using "Tracker" when the ball finishes the motion on the inclined surface. According to the instruction of the Tracker, the moment tracking starts would be the $t = 0$. Extract the horizontal ($x(t)$)and vertical ($y(t)$) components of the position as a function of time, *t*.

5.1.5 *Fit a function*: According to the **Eqs. 1 and 2** and based on the discussion in the 'Free fall lab', the motion in *x* has constant velocity while the motion in *y* direction has constant acceleration. According to the instruction for **Plotting and Fitting**, fit an a linear function for *x* and quadratic function for *y* components, separately.

5.1.6 *Parameters*: When fitting a function, each parameter corresponds to a physical quantity (see **Eq. 2**). For example, when fitting $A + Bt$ to $x(t)$, $B$ corresponds to $v_0 \cos\theta$ and $A$ to $x_0$. Similarly, think of the physical meaning of the parameters for the fitted quadratic function to $y(t)$ and fill in **Table. 1**.

5.1.7 *Velocity*: The instantaneous velocity can be estimated from the extracted positions as following,

$$\begin{cases} v_x(t_i) = \frac{x(t_{i+m})-x(t_{i-m})}{t_{i+m}-t_{i-m}} \\ v_y(t_i) = \frac{y(t_{i+m})-y(t_{i-m})}{t_{i+m}-t_{i-m}}, \end{cases} \tag{4}$$

where $m \geq 1$. Estimate velocities for $m = 1, m = 2$ and plot them as a function of time, *t*.

## 5.2 Bouncing motion ($\theta_0 > 0$)

5.2.1 *Perform the experiment*: Move the inclined surface away from the edge of the table by at least 5 cm. When released, the ball should bounce from the table. According to 5.1.3, Perform the experiment at least 5 times with different initial angles of the inclined plane.

5.2.2 *Fit functions*: Similar to part 5.1, fit appropriate functions to $x(t)$ and $y(t)$ and determine $v_{x0}(v_0 \cos\theta_0)$ and $v_{y0}(v_0 \sin\theta_0)$. You should only fit the functions to the motion after bouncing. Estimate the magnitude of the initial velocity ($v_0$) and the initial angle ($\theta_0$) of the projectile motion.

5.2.3 *Angle of inclined surface*: Measure the inclined surface's angle using the "Tracker"or alternatively, measure it using a protractor. Given that bouncing from the table is perpendicular to the motion on the inclined surface, think of how the angle of the incline surface and initial angle for the bouncing motion are related.

5.2.4 *Maximum height*: In the bouncing motion, the ball will reach a certain maximum height with respect to the table. At that point, the vertical velocity $(v_y)$ would become zero. Use **Eq. 2** to find the time the ball reaches the maximum height $(t_H)$. Then, substitute $t_H$ in **Eq. 3** and derive the expression for the maximum height $(H)$ with respect to the table.

5.2.5 *Comparison with Tracker*: According to the expressions in 5.2.4, estimate $t_H$ and $H$ for 5 trials and fill in the rest of the **Table. 2**. Height can also be directly measured in the tools provided in the "Tracker". Compare the height measurement from "Tracker" to the analytical values from 5.2.4 and fill in the rest of **Table. 2**.

5.2.6 *Range of motion*: As illustrated in **Fig. 2**, the ball at a certain time $(t_R)$ will reach the same level as when it bounced from the table. The $t_R$ can be estimated by assuming $y(t_R) = y_0$ in **Eq. 3**. The range of motion $(R)$ (see **Fig. 2**) is the horizontal displacement of the object from the beginning of the bouncing motion $(t = 0)$ until reaching the same level $(t = t_R)$ which can be estimated from $x(t_R)$. Derive the expressions for $t_R$ and $x(t_R)$.

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# Lab II Worksheet

**Name of the Student:** ______________________ **Date:** _____________________

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## Q.1 Downward projectile (see Instruction Section 5.1 for $\theta_0 < 0$)

Initial angle: ------------------

*Q.1.1* Plot the $y(t)$ and $x(t)$ as a function of time for 5 different trials. Describe the differences between the vertical and horizontal motion

*Q.1.2* According to Eq. 2, what type of functions are appropriate to fit to the vertical and horizontal components of the motion? Explain.

*Q.1.3* Fill in the table according to the fitted parameters to $x(t)$ and $y(t)$.

**Table. 1**

| Trial | $x_0(m)$ | $y_0(m)$ | $v_{x0}(\frac{m}{s})$ | $v_{y0}(\frac{m}{s})$ | $g(\frac{m}{s^2})$ |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| Average | | | | | |

*Q.1.4* Based on the average value of $v_{x0}$ and $v_{y0}$, estimate the magnitude of the initial velocity ($v_0$) and the initial angle ($\theta_0$) from the averaged values.

*Q.1.5* Compare the direct measurement of the initial angle with the one calculated in *Q.1.4*.

*Q.1.6* According to **Eq. 4** calculate the velocity for *x*-component ($v_x$) and *y*-component ($v_y$) for $m = 1$ and $m = 2$ and plot them as a function of time, *t* for only trial.

## Q.2 Bouncing motion (see Instruction Section 5.2 for $\boldsymbol{\theta_0 > 0}$)

The following questions are for the bouncing motion. Perform the experiment at 5 different incline plane's angles.

*Q.2.1* Fit appropriate functions to vertical and horizontal components of the motion and fill in the first 5 columns of the below table. (*Hint: Follow the same steps as in Q.1*)

**Table. 2**

| Trial | Inclined angle | $v_{x0}$ | $v_{y0}$ | $\theta_0$ | $t_H$ | $H$ | $H(Tracker)$ | Percent Error |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | | | |

*Q.2.2* What is the relation between the angle of the incline surface and the initial angle ($\theta_0$)?

*Q.2.3* Derive the equations for the time $t_H$, the time it takes for the ball to reach maximum height.

*Q.2.4* Use the equations in *Q.2.3* to fill in columns 6 and 7 of the table. Afterwards, use "Tracker" to estimate the height directly and fill in column 8. What is the percent difference?

*Q.2.5* After reaching the maximum height, how long does it take for the ball to reach the height of the table's surface again? Discuss your reasoning. (*Hint: The answer should be in terms of* $t_H$)

*Q.2.6* What are $v_x$ and $v_y$ of the ball after reaching the same level as the table? (*Hint: see Fig-2*)

*Q.2.7* According to the instructions in 5.2.6, derive the expressions for $t_R$ and $R$ (range of the motion).

*Q.2.8* According to the expressions in *Q.2.7*, at what initial angle would the ball has the maximum range? Discuss your reasonings.